Active Network Matching Of Arbitrary Loads

by

R. A. Rohrer

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ABSTRACT

Passive matching of an arbitrary load to a purely resistive source is limited both as to obtainable gain-bandwidth product and as to the practicality of the passive network required. In this paper, a matching technique which uses one active element, a negative impedance converter, in a lossless coupling network is presented. The ideal match achieved is not frequency limited, and the matching network consists of only a small number of elements. The only limitation on the method is that the load immittance must be either minimum susceptive or minimum reactive.

I INTRODUCTION

Existing methods of matching arbitrary loads to purely resistive sources by means of lossless coupling networks are severely limited. The reflection coefficient limitation theorem developed by Bode[1] has been extended by Fano[2] to give the theoretical limitation on the maximum obtainable gain-bandwidth product when matching an arbitrary load. The knowledge of this limitation, however, does not notably facilitate the design of the lossless coupling network.

The inclusion of one ideal active element, a negative impedance converter (NIC),[3] into the coupling network can result in an optimum match which is subject to no frequency limitations. Moreover, the matching network to be obtained is easily realizable with a small number of elements.

II DERIVATION OF THE GENERAL MATCHING NETWORK

A. Darlington's Synthesis

The arbitrary load impedance to be matched can be specified in the usual manner by

$$Z_L(s) = \frac{n_1 + n_1}{m_2 + n_2} \qquad (1)$$

It is always possible to construct the Darlington equivalent circuit,[4] consisting of a lossless two-port network terminated in a one ohm resistance (see Fig. 1) by making the identifications

$$y_{11} = \frac{m_1}{n_2};\ y_{12} = \frac{m_2}{n_2};\ z_{22} = \frac{m_1}{n_1}\ ;\ y_{12} = \frac{\sqrt{m_1 m_2 - n_1 n_2}}{n_2} \quad \text{Case A} \qquad (2a)$$

or

$$y_{11} = \frac{n_1}{m_2}\ ;\ y_{22} = \frac{n_2}{m_2};\ z_{22} = \frac{n_1}{m_1};\ y_{12} = \frac{\sqrt{n_1 n_2 - m_1 m_2}}{m_2} \quad \text{Case B} \qquad (2b)$$

The two cases are mutually exclusive, and a rational y_{12} is always obtainable by suitable augmentation of the original function.[4]

B. The Constant Resistance Lattice

A classic network which provides an optimum power match between two one ohm resistance loads for all real frequencies is the constant resistance lattice of Fig. 2. The optimum match is contingent upon the series and shunt arms being lossless and maintaining the impedance relationship

$$z_a = \frac{1}{z_b} \qquad (3)$$

We may relate the lattice to the Darlington equivalent network by making the identifications

$$z_a = \frac{1}{z_{22}} \qquad (4a)$$

$$z_b = z_{22} \qquad (4b)$$

This lattice, pictured in Fig. 3a, can be unbalanced by the usual series and shunt removal methods,[4] as shown in successive steps in Fig. 3. We may see in Fig. 3d that the left and right hand networks are each the Darlington equivalent network of the given load (see Fig. 1). The network on the right can be replaced by that load, and the terminal resistance on the left can be considered to be the source resistance.

The two identical networks in the upper and lower series arms can be combined in the upper arm. The impedance for this combination is given for either Case A or Case B, Eq. (2), by

$$Z_{ab}(s) = 2\,\frac{m_1 n_2 - m_2 n_1}{m_2^2 - n_2^2} \tag{5}$$

Eq. (5) is an expression for twice the negative of the odd part of the original load impedance; hence,

$$Z_{ab}(s) = -\,2\ \mathrm{OD}Z_L(s) \tag{6}$$

Therefore, the realization of an optimum matching network is seen to be related to the problem of synthesizing $-2\mathrm{OD}Z_L(s)$. A dual analysis, beginning with Eq. (1), would yield a comparable result on the admittance basis. The optimum matching network is shown on both bases in Fig. 4.

Stability, usually a moot point in active network synthesis, is guaranteed because of the equivalence of the final network to the entirely passive constant resistance lattice. A method for realizing $-2\mathrm{OD}Z_L(s)$ is the only remaining obstacle to the complete synthesis.

III IMMITTANCE ODD PART REALIZATION

A. Realizability of Odd Part

The synthesis of -2ODZ(s) (or -2ODY(s)) is best approached through obtaining a realization for the positive odd part. Consider the impedance

$$Z(s) = \frac{m_1 + n_1}{m_2 + n_2} \qquad (7)$$

Its odd part is given by

$$ODZ(s) = \frac{m_2 n_1 - m_1 n_2}{m_2^2 - n_2^2} \qquad (8)$$

This expression can be factored to obtain

$$ODZ(s) = \frac{m_1}{n_2} \frac{\dfrac{n_1}{m_1} - \dfrac{n_2}{m_2}}{\dfrac{m_2}{n_2} - \dfrac{n_2}{m_2}} \qquad \text{Case A} \quad (9a)$$

or

$$ODZ(s) = \frac{n_1}{m_2} \frac{\dfrac{m_1}{n_1} - \dfrac{m_2}{n_2}}{\dfrac{n_2}{m_2} - \dfrac{m_2}{n_2}} \qquad \text{Case B} \quad (9b)$$

The cascade structure shown in Fig. 5 has the input impedance

$$Z_{11}(s) = z_{11} \frac{\dfrac{1}{y_{22}} - Z'_L}{z_{22} - Z'_L} \qquad (10)$$

For the present purposes, it will suffice to state that the NIC transforms the load at port two so that the negative of the load is presented at port one. Comparison of Eqs. (9) and (10) yields the relations

$$z_{11} = \frac{m_1}{n_2};\ z_{22} = \frac{m_2}{n_2};\ y_{22} = \frac{m_1}{n_1};\ Z_L' = \frac{n_2}{m_2} \qquad \text{Case A} \qquad (11a)$$

or

$$z_{11} = \frac{n_1}{m_2};\ z_{22} = \frac{n_2}{m_2};\ y_{22} = \frac{n_1}{m_1};\ Z_L' = \frac{m_2}{n_2} \qquad \text{Case B} \qquad (11b)$$

The parameters of the network N, z_{11}, z_{22}, and y_{22}, are recognized to be the duals of the corresponding parameters for the Darlington equivalent network (see Eqs. (2)). Therefore, we may state that

$$z_{12} = \frac{\sqrt{m_1 m_2 - n_1 n_2}}{n_2}, \qquad \text{Case A} \qquad (12a)$$

or

$$z_{12} = \frac{\sqrt{n_1 n_2 - m_1 m_2}}{m_2}, \qquad \text{Case B} \qquad (12b)$$

Furthermore, the load impedance, Z_L', is in both cases an LC driving point function. The Darlington realization is always possible; therefore, the above realization of ODZ(s) is always possible.

The major problem which arises in ODZ(s) synthesis is that of

insuring a rational z_{12}, which, however, can always be accomplished by augmenting the given function. Assume that a given impedance odd part determines a z_{12}^2 of Case A, Eq. (12a), for which the numerator is not a full square. The numerator should be multiplied by the even factor $(m_0^2-n_0^2)$ which makes it a full square. To perform this numerator multiplication without altering the original function, we must perform a similar multiplication on the denominator, which amounts to changing the original impedance to

$$Z(s) = \frac{(m_1 + n_1)}{(m_2 + n_2)} \times \frac{(m_0 + n_0)}{(m_0 + n_0)} \tag{13}$$

The corresponding change on the odd part, which also maintains its original value, is

$$ODZ(s) = \frac{(m_2 n_1 - m_1 n_2)}{(m_2^2 - n_2^2)} \times \frac{(m_0^2 - n_0^2)}{(m_0^2 - n_0^2)} \tag{14}$$

Thus even and odd components must be revised:

$$\begin{aligned} m_1' &= m_1 m_0 + n_1 n_0 \\ n_1' &= n_1 m_0 + m_1 n_0 \\ m_2' &= m_2 m_0 + n_2 n_0 \\ n_2' &= n_2 m_0 + m_2 n_0 \end{aligned} \tag{15}$$

These new relations may be used in Eqs. (11a) and (12a) to obtain realizable network parameters, including a rational z_{12}. A similar procedure for networks of Case B will also yeild a rational z_{12} and a comparable set of parameters.

B. Practical Realization of Odd Part

A source with one ohm internal resistance (as is implied in Fig. 4) is not often desired. Any resistance, R, can be accomodated by performing a loop impedance level change on the Darlington network (i.e., multiplying z_{22} by R and z_{12} by $\sqrt{R}$).

This same type of impedance level change can be applied to the realization of the odd part to eliminate the need for ideal transformers in that portion of the matching network.[5] The input impedance expression, Eq. (10), can be rewritten in the equivalent form,

$$Z_{11}(s) = z_{11} - \frac{z_{12}^2}{z_{22} - Z_L'} \tag{16}$$

Eq. (16) is also equivalent to

$$Z_{11}(s) = z_{11} - \frac{(Kz_{12})^2}{K^2 z_{22} - K^2 Z_L'} \tag{17}$$

The first step in the procedure is to synthesize z_{11}, satisfying the zeros of z_{12}, which will actually yield the other network parameters $z_{22}' = Kz_{12}$ and z_{22}'. The final step is to factor the now required output loop impedance expression into a sum of positive and negative components, i.e.,

$$K^2 z_{22} - K^2 Z_L' - z'_{22} = Z^+ - Z^- \tag{18}$$

The final transformerless realization of ODZ(s) takes the form given in Fig. 6.

C. Realization of $-2ODZ_L(s)$.

The method of obtaining a realization for twice the negative of the odd part of the given load impedance is to equate it, whenever possible, to the odd part of another positive real impedance, i.e.,

$$-2ODZ_L(s) = ODZ'(s) \tag{19}$$

To ascertain the validity of this relation, we must investigate the restrictions on the odd parts of positive real immittances. These conditions may be readily obtained from the restrictions on positive real functions, and are found to be:

i) The degree of the numerator must be less than or equal to one greater than that of the denominator. (20a)

ii) All $j\omega$ -axis poles must be of single order and have real and positive residues. (20b)

Since the odd part, ODZ'(s), is equated to the negative of another odd part, the first of conditions (20) is automatically satisfied. However, the second may only be satisfied when there are no $j\omega$ -axis poles, as positive residues for those of $ODZ_L(s)$ would imply negative residues for the corresponding poles of ODZ'(s). In other words, Eq. (19) is valid for minimum reactive impedances only. A dual approach would show that $-2ODY_L(s)$ is realizable with one NIC for minimum susceptive admittances, $Y_L(s)$.

Therefore, an arbitrary load immittance which is either minimum

reactive or minimum susceptive can be matched as in Fig. 4. Since most driving-point immittances are usually one or the other, a single NIC matching network is almost always possible. If a given function is neither minimum reactive nor minimum susceptive, or if it is more desirable to use a particular basis (Fig. 4a or 4b), the load can always be made to satisfy the necessary requirement by adding a suitable lossless network in series, parallel, or cascade with it.

IV ILLUSTRATIVE EXAMPLES

The above theory may be applied to realize an optimum match to the load impedance of Fig. 7,

$$Z_L(s) = \frac{1+s}{1+2s} \tag{21}$$

This minimum reactive impedance has as its odd part

$$ODZ_L(s) = \frac{-s}{1-4s^2}$$

For a match on the impedance basis, we must realize

$$ODZ'(s) = \frac{2s}{1-4s^2}$$

The obvious realization of this odd part is combined with the Darlington equivalent network for $Z_L(s)$ to give the optimum match shown in Fig. 8.

As a final example, we can consider the load of Fig. 9, specified by

$$Z_L(s) = \frac{s^2+s+1}{s+1} \tag{22}$$

This impedance is not a minimum reactive function; however, its reciprocal

is minimum susceptive,

$$Y_L(s) = \frac{s+1}{s^2+s+1}$$

The odd part of this admittance is

$$ODY_L(s) = \frac{s^3}{s^4+s^2+1}$$

Therefore, the admittance to be realized is

$$ODY'(s) = \frac{-2s^3}{s^4+s^2+1}$$

In the second order case, we can always obtain a rational y_{12} without augmentation, because the arbitrary constant of the even part allows one extra degree of freedom in determining the positive real function which is associated with the given odd part. The admittance is most generally [5]

$$Y'(s) = \frac{a_3s^3+a_2s^2+a_1s+a_0}{s^2+s+1}$$

The above odd part requires that

$$a_3 = 0$$
$$a_1 - a_2 = 2$$
$$a_1 \quad a_0 = 0$$

If we are to obtain a rational y_{12}, the following equation must hold:

$$(a_2 + a_0 - a_1)^2 = 4\, a_0\, a_2$$

Solution of this relations yields

$$a_0 = 2/3$$
$$a_1 = 2/3$$
$$a_2 = 8/3$$
$$a_3 = 0$$

Hence, the admittance is given by

$$Y'(s) = \frac{1}{3} \quad \frac{8s^2 + 2s + 2}{s^2 + s + 1}$$

It is of Case A, and the parameters which realize the odd part are

$$y_{11} = \frac{8/3\ s^2 + 2/3}{s},$$

$$y_{12} = \frac{\pm\sqrt{2/3}\,(2s^2 + 1)}{s},$$

$$y_{22} = \frac{s^2 + 1}{s},$$

$$Y'_L = \frac{s}{s^2 + 1}$$

A transformerless realization of this odd part, obtained by the procedure outlined in section III. B, is shown in Fig. 10. The matching network is enclosed in dashes in the final realization shown in Fig. 11.

V CONCLUSION

The use of one active element in the coupling network has rendered a general method of optimally matching an arbitrary load to a purely resistive source. The matching network obtained is practical, requiring in general only a few more elements than are required to represent the load. No frequency limitations are encountered, except, of course, those practical limitations attendant upon the devices being employed. The most severe limitation encountered is that the load must be either minimum reactive or minimum susceptive, but any load can be made so by proper supplementation with lossless elements.

ACKNOWLEDGMENT

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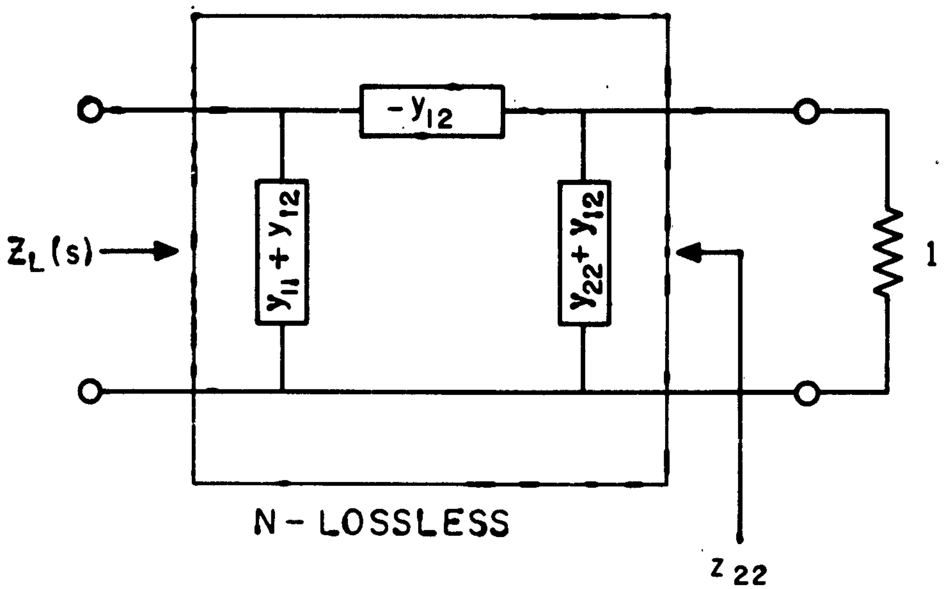


Fig. 1--Darlington Equivalent of $Z_L(s)$

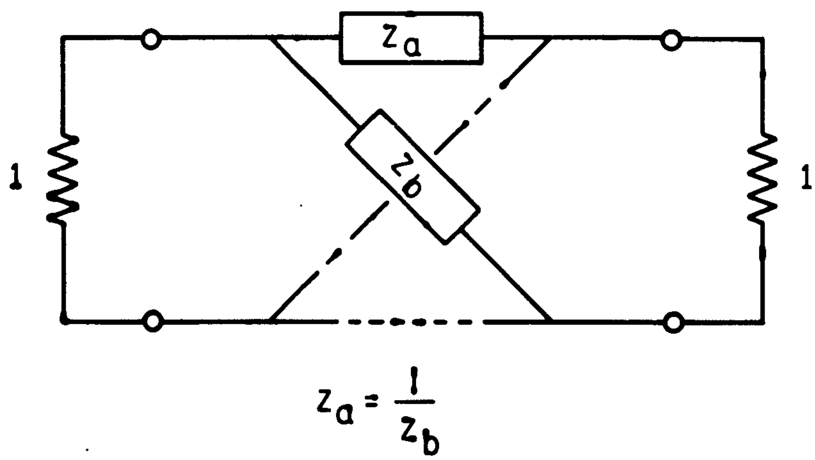
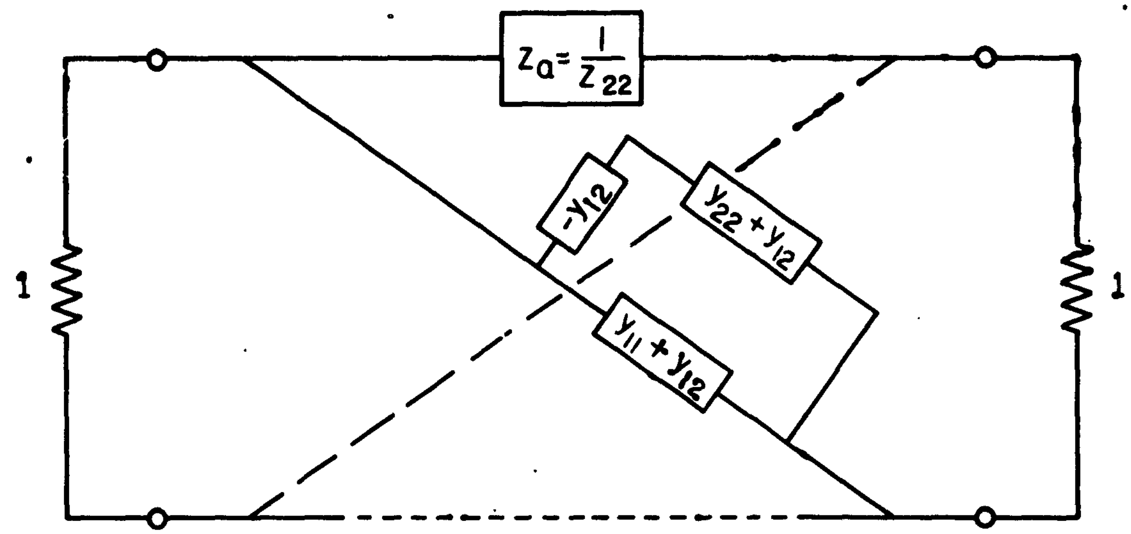
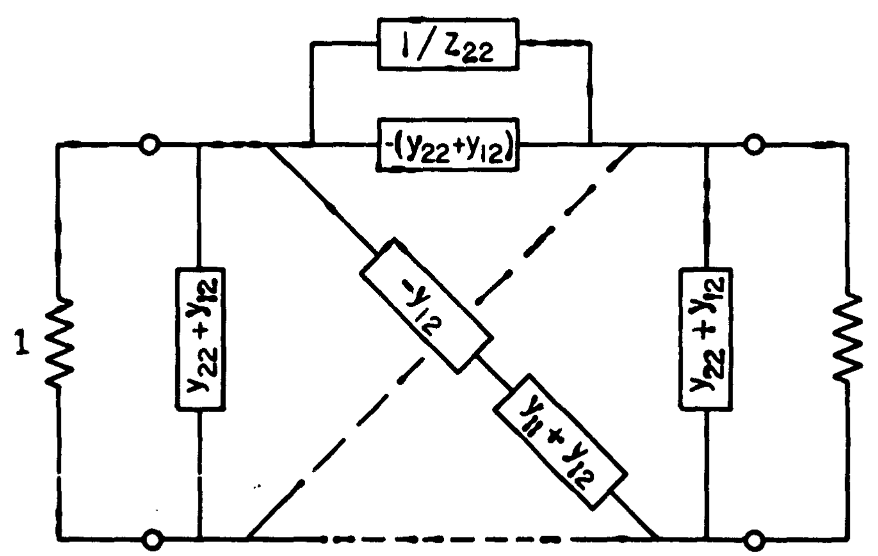


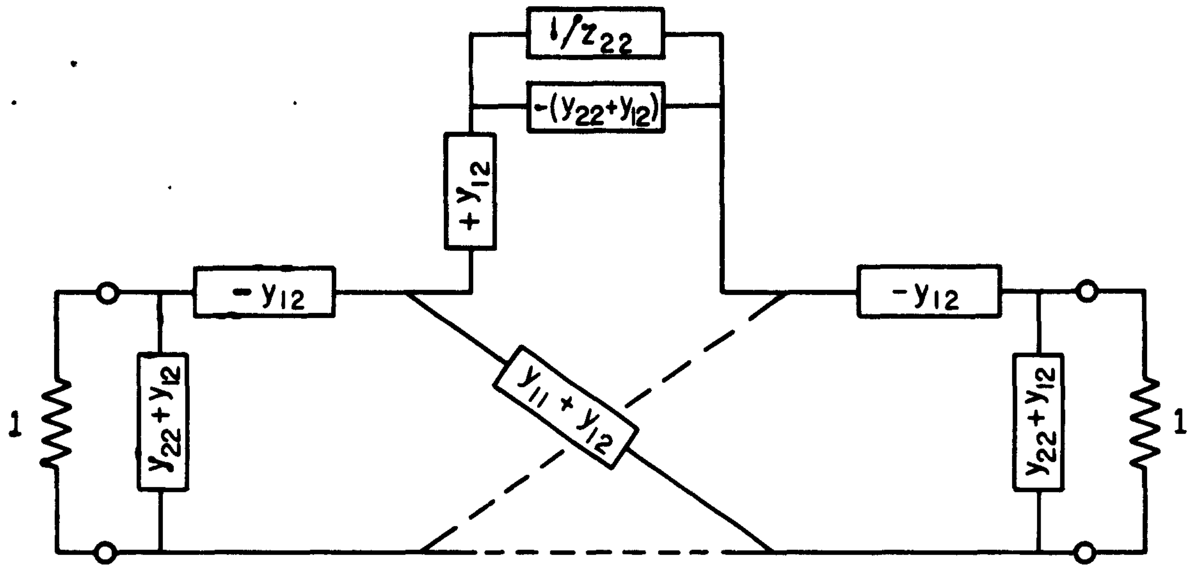
Fig. 2--Constant Resistance Lattice Match



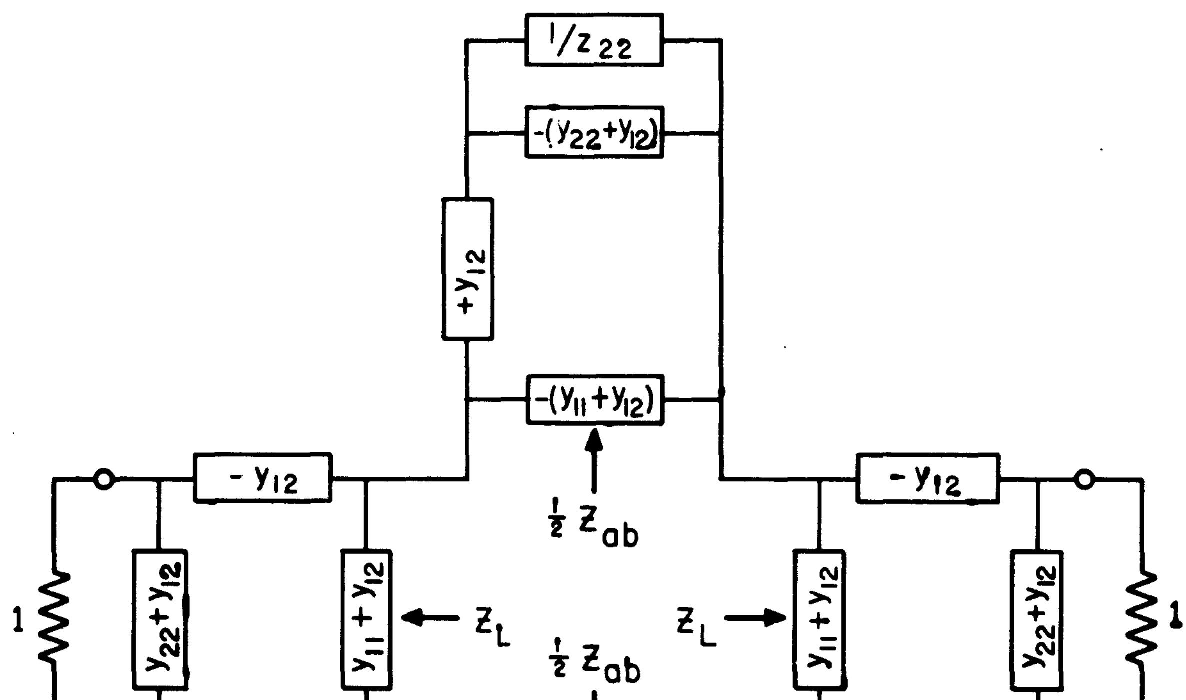
a) Lattice Match



b) Result of Shunt Unbalancing

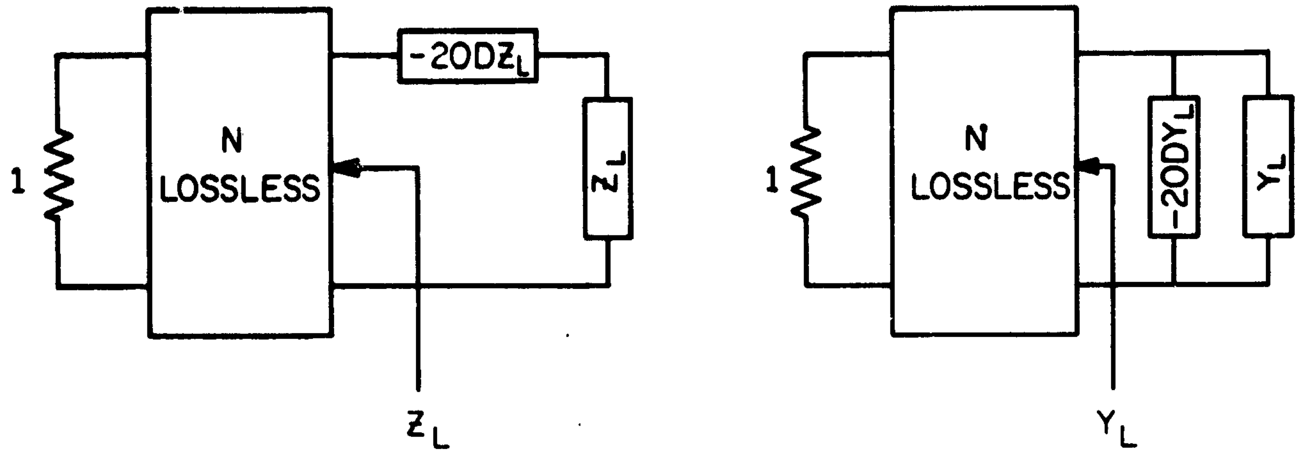


c) Result of Shunt, Then Series Unbalancing



d) Final Form

Fig. 3--Unbalancing of Constant Resistance Lattice



a) Z-Basis Match

b) Y-Basis Match

Fig. 4--Optimum Match

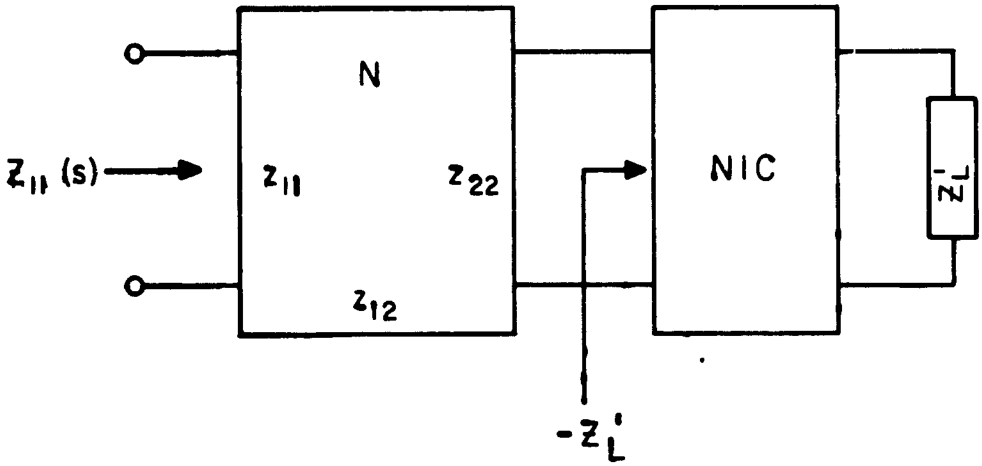


Fig. 5--Cascade Structure for Realization of ODZ(s)

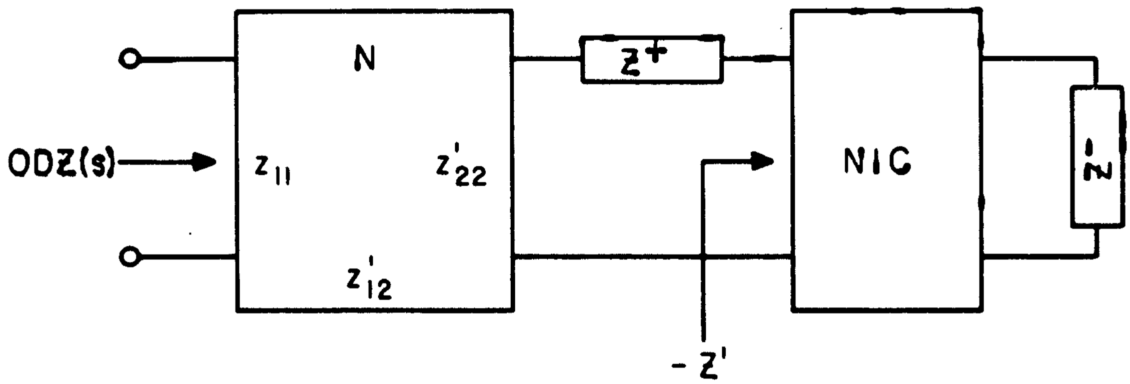


Fig. 6--Transformerless Realization of ODZ(s)

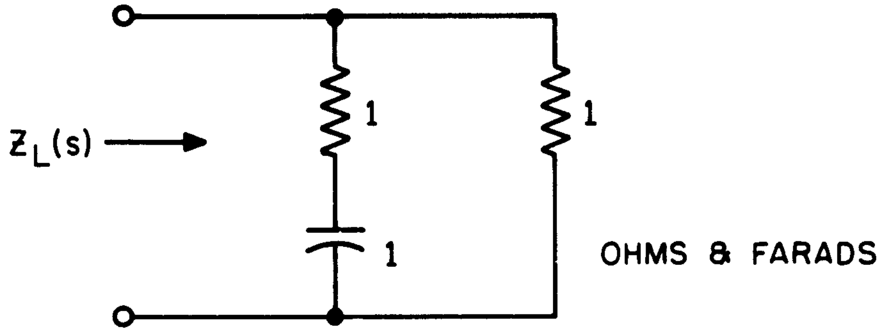


Fig. 7--Load Impedance, $Z_L(s) = \frac{1+s}{2+s}$

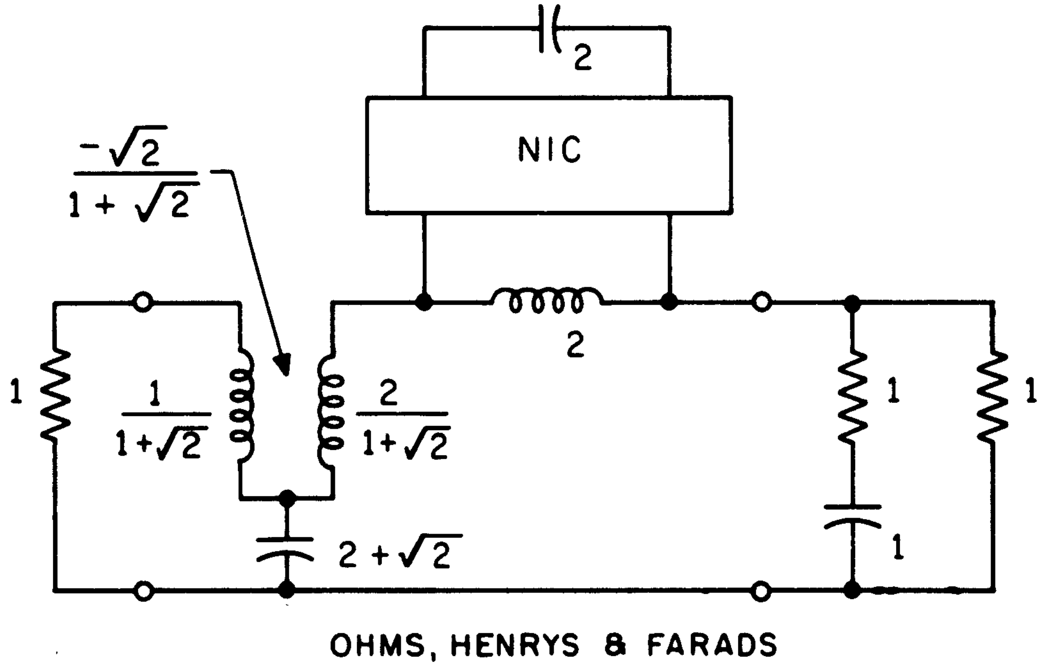


Fig. 8--Optimum Match to $Z_L(s) = \frac{1+s}{2+s}$

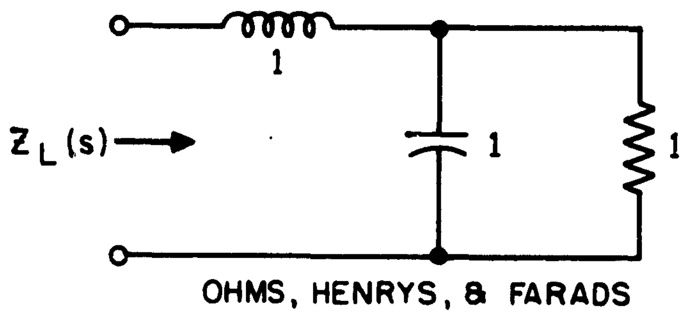


Fig. 9--Load Impedance, $Z_L(s) = \frac{s^2 + s + 1}{s + 1}$

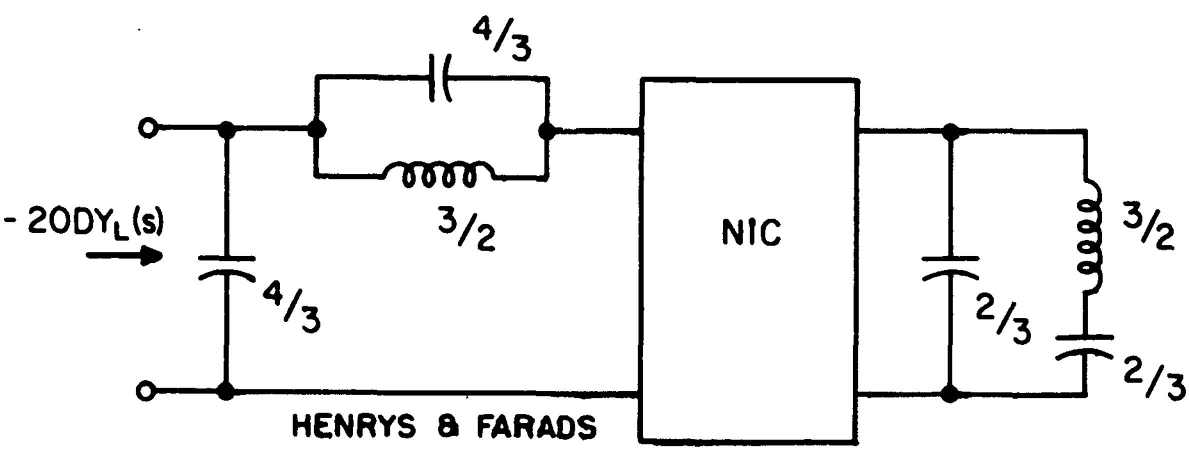


Fig. 10--Transformerless Realization of

$$-20DY_L(s) = \frac{-2s^3}{s^4 + s^2 + 1}$$

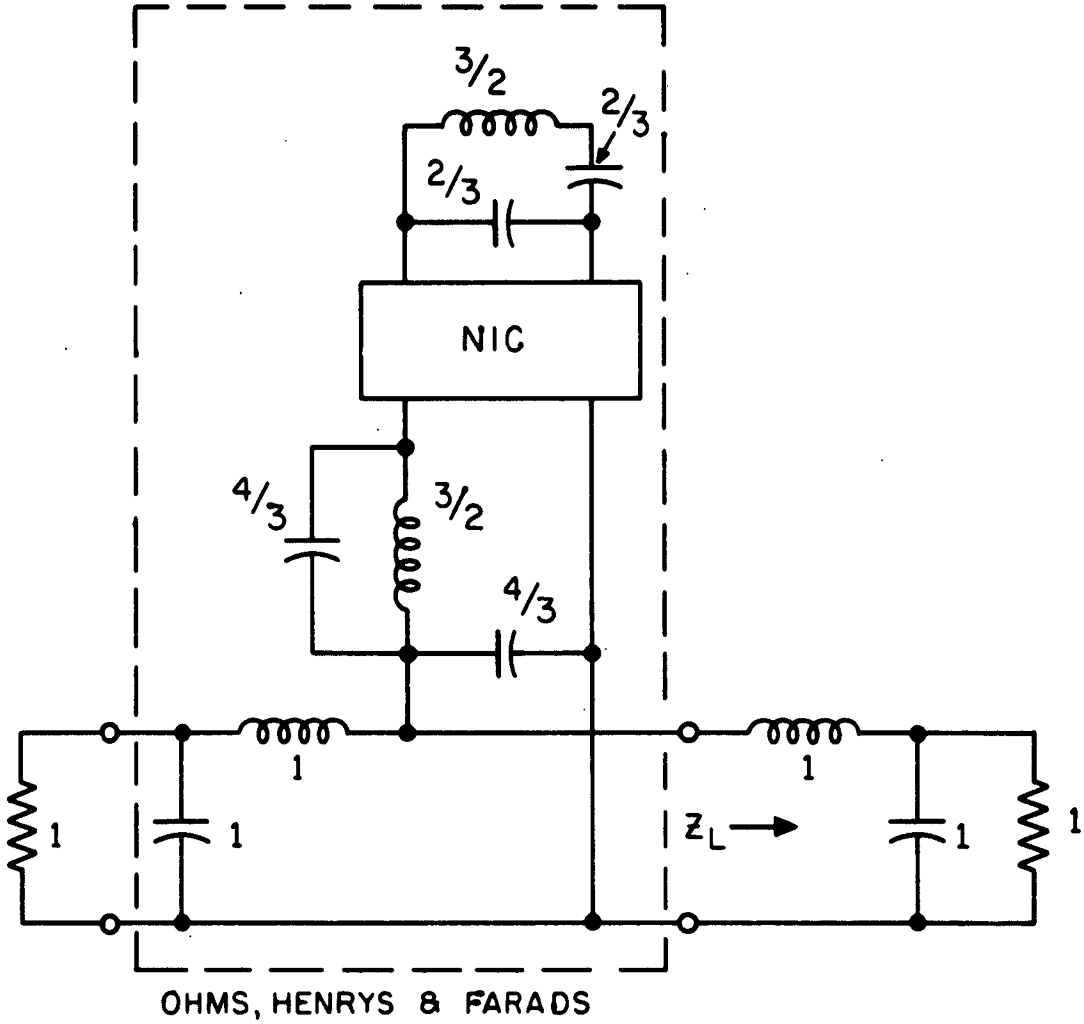


Fig. 11--Optimum Match to $Z_L(s) = \frac{s^2 + s + 1}{s + 1}$

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